Military Reliability Modeling

William P. Fox, Steven B. Horton

Introduction

You are an infantry rifle platoon leader. Your platoon is occupying a battle position and has been ordered to establish an observation post (OP) on a hilltop approximately one kilometer forward of your position. The OP will be occupied by three soldiers for 24 hours. Hourly situation reports must be made by radio. All necessary rations, equipment, and supplies for the 24 hour period must be carried with them. The OP is ineffective unless it can communicate with you in a timely manner. Therefore, radio communications must be reliable. The radio has several components which affect its reliability, an essential one being the battery. Batteries have a useful life which is not deterministic (we do not know exactly how long a battery will last when we install it). Its lifetime is a variable which may depend on previous use, manufacturing defects, weather, etc. The battery that is installed in the radio prior to leaving for the OP could last only a few minutes or for the entire 24 hours. Since communications are so important to this mission, we are interested in modeling and analyzing the reliability of the battery.

We will use the following definition for reliability:

If T is the time to failure of a component of a system, and $f(t)$ is the probability distribution function of T, then the components' *reliability* at time t is

$$R(t) = P\,(T > t) = 1 - F(t).$$

$R(t)$ is called the reliability function and $F(t)$ is the cumulative distribution function of $f(t)$.

A measure of this reliability is the probability that a given battery will last more than 24 hours. If we know the probability distribution for the battery life, we can use our knowledge of probability theory to determine the reliability. If the battery reliability is below acceptable standards, one solution is to have the soldiers carry spares. Clearly, the more spares they carry, the less likely there is to be a failure in communications due to batteries. Of course the battery is only one component of the radio. Others include the antenna, handset, etc. Failure of any one of the essential components causes the system to fail.

This is a relatively simple example of one of many military applications of reliability. This chapter will show we can use elementary probability to generate models that can be used to determine the reliability of military equipment.

Modeling Component Reliability

In this section, we will discuss how to model component reliability. Recall that the reliability function, $R(t)$ is defined as:

$$R(t) = P(T > t) = P(\text{component fails after time } t).$$

This can also be stated, using T as the component failure time, as

$$R(t) = P(T > t) = 1 - P(T \le t) = 1 - \int_{-\infty}^{t} f(x)dx = 1 - F(t).$$

Thus, if we know the probability density function $f(t)$ of the time to failure T, we can use probability theory to determine the reliability function $R(t)$. We normally think of these functions as being time dependent; however, this is not always the case. The function might be discrete such as the lifetime of a cannon tube. It is dependent on the number of rounds fired through it (a discrete random variable).

A useful probability distribution in reliability is the exponential distribution. Recall that its density function is given by

$$f(t) = \begin{cases} l e^{-lt} & t > 0 \\ 0 & \text{otherwise} \end{cases},$$

where the parameter l is such that $1/l$ equals the mean of the random variable T. If T denotes the time to failure of a piece of equipment or a system, then $1/l$ is the mean time to failure which is expressed in units of time. For applications of reliability, we will use the parameter l. Since $1/l$ is the mean time to failure, l is the average number of failures per unit time or the failure rate. For example, if a light bulb has a time to failure that follows an exponential distribution with a mean time to failure of 50 hours, then its failure rate is 1 light bulb per 50 hours or 1/50 per hour, so in this case $l = 0.02$ per hour. Note that the mean of T, the mean time to failure of the component, is $1/l$.

Example 1: Let's consider the example presented in the introduction. Let the random variable T be defined as follows:

T = time until a randomly selected battery fails.

Suppose radio batteries have a time to failure that is exponentially distributed with a mean of 30 hours. In this case, we could write

$$T \sim \text{EXP}\left(l = \frac{1}{30} \right).$$

Therefore, $l = \frac{1}{30}$ per hour, so that

$$f(t) = \frac{1}{30} e^{\frac{-1}{30}t},\ t > 0 \text{ and } F(t) = \int_0^t \frac{1}{30} e^{\frac{-1}{30}x} dx .$$

$F(t)$, the CDF of the exponential distribution, can be integrated to obtain

$$F(t) = 1 - e^{\frac{-1}{30}t},\ t > 0 .$$

Now we can compute the reliability function for a battery:

$$R(t) = 1 - F(t) = 1 - \left(1 - e^{\frac{-1}{30}t} \right) = e^{\frac{-1}{30}t},\ t > 0 .$$

Recall that in the earlier example, the soldiers must occupy the OP for 24 hours. The reliability of the battery for 24 hours is

$$R(24) = e^{\frac{-1}{30}(24)} = 0.4493 ,$$

so the probability that the battery lasts more than 24 hours is 0.4493.

Example 2: We have the option to purchase a new nickel cadmium battery for our operation. Testing has shown that the distribution of the time to failure can be modeled using a parabolic function:

$$f(x) = \begin{cases} \left(\frac{x}{384} \right)\left(1 - \frac{x}{48} \right) & 0 \le x \le 48 \\ 0 & \text{otherwise} \end{cases}$$

Let the random variable T be defined as follows:

T = time until a randomly selected battery fails.

In this case, we could write

$$f(t) = (t/384)(1 - t/48),\ 0 \le t \le 48 \text{ and } F(t) = \int_0^t (x/384)(1 - x/48)dx .$$

Recall that in the earlier example the soldiers must man the OP for 24 hours. The reliability of the battery for 24 hours is therefore

$$R(24) = 1 - F(24) = 1 - \int_0^{24} (t/384)(1 - t/48)dx = 0.500$$

which is an improvement over the batteries from example 1.

Modeling Series Systems

Now we consider is a system with n components $C_1, C_2, \ldots, C_n$ where each of the individual components must work in order for the system to function. A model of this type of system is shown in figure 1.

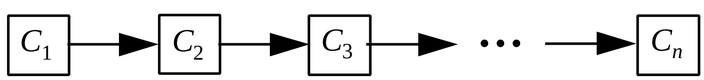


Figure 1: Series system

If we assume these components are mutually independent, the reliability of this type of system is easy to compute. We denote the reliability of component i at time t by $R_i(t)$. In other words, $R_i(t)$ is simply the probability that component i will function continuously from time 0 through until time t. We are interested in the reliability of the entire system of n components, but since these components are mutually independent, the system reliability is

$$R(t) = R_1(t) \cdot R_2(t) \cdot \ \cdots \ \cdot R_n(t) .$$

Example 3: Our radio has several components. Let us assume that there are four major components -- they are (in order) the handset, the battery, the receiver-transmitter, and the antenna. Since they all must function properly for the radio to operate, we can model the radio with the diagram shown in figure 2.

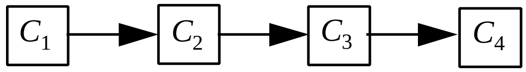


Figure 2: Radio System

Suppose we know that the probability that the handset will work for at least 24 hours is 0.6703, and the reliabilities for the other components are 0.4493, 0.7261, and 0.9531, respectively. If we assume that the components work *independently* of each other, then the probability that the entire system works for 24 hours is:

$$R(24) = R_1(24) \cdot R_2(24) \cdot R_3(24) \cdot R_4(24) = (.6703)(.4493)(.7261)(.9531) = 0.2084 .$$

Recall that two events *A* and *B* are *independent* if $P(A|B) = P(A)$.

Modeling Parallel Systems (Two Components)

Now we consider a system with two components where only one of the components must work for the system to function. A system of this type is depicted in figure 3.

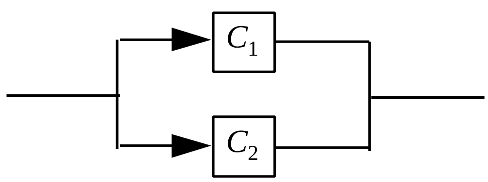


Figure 3: Parallel System of Two Components

Notice that in this situation the two components are *both* put in operation at time 0; they are both subject to failure throughout the period of interest. Only when *both* components fail before time *t* does the system fail. Again we also assume that the components are independent. The reliability of this type of system can be found using the following well known model:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

In this case, *A* is the event that the first component functions for longer than some time, *t*, and *B* is the event that the second component functions longer than the same time, *t*. Since reliabilities *are* probabilities, we can translate the above formula into the following:

$$R(t) = R_1(t) + R_2(t) - R_1(t)R_2(t).$$

Example 4: Suppose your battalion is crossing a river that has two bridges in the area. It will take 3 hours to complete the crossing. The crossing will be successful as long as at least one bridge remains operational during the entire crossing period. You estimate that enemy guerrillas with mortars have a one-third chance of destroying bridge 1 and a one-fourth chance of destroying bridge 2 in the next 3 hours. Assume the enemy guerrillas attacking each bridge operate independently. What is the probability that your battalion can complete the crossing?

Solution: First we compute the individual reliabilities:

$$R_1(3) = 1 - \frac{1}{3} = \frac{2}{3}$$

and

$$R_2(3) = 1 - \frac{1}{4} = \frac{3}{4}.$$

Now it is easy to compute the system reliability:

$$R(3) = R_1(3) + R_2(3) - R_1(3)R_2(3) = \frac{2}{3} + \frac{3}{4} - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = \frac{11}{12} = 0.9167.$$

Modeling Active Redundant Systems

Consider the situation in which a system has n components, all of which begin operating (are active) at time $t = 0$. The system continues to function properly as long as at least k of the components do **not** fail. In other words, if $n - k + 1$ components fail, the system fails. This type of component system is called an **active redundant system**. The active redundant system can be modeled as a **parallel** system of components as shown in figure 4 below:

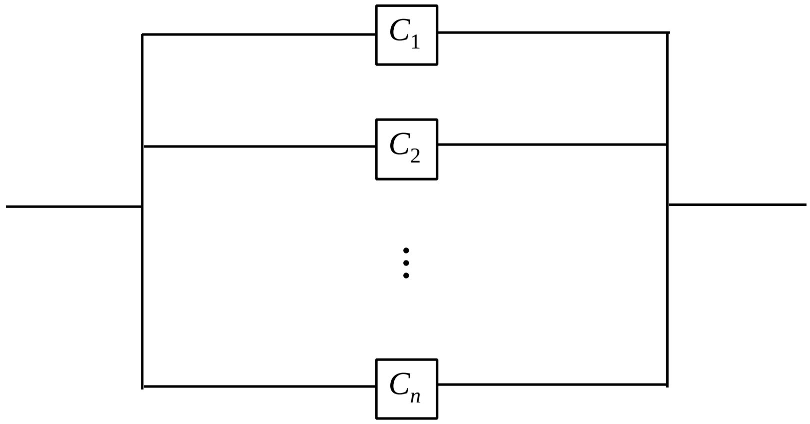


Figure 4: Active Redundant System

We assume that all n components are identical and will fail independently. If we let T_i be the time to failure of the ith component, then the T_i terms are independent and identically distributed for $i = 1,2,3,\ldots,n$. Thus $R_i(t)$, the reliability at time t for component i, is identical for all components.

Recall that our system operates if at least k components function properly. Now we define the random variables X and T as follows:

X = number of components functioning at time t, and
T = time to failure of the entire system.

Then we have

$$R(t) = P(T > t) = P(X \geq k).$$

It is easy to see that we now have n identical and independent components with the same probability of failure by time t. This situation corresponds to a binomial experiment and we can solve for the system reliability using the binomial distribution with parameters n and $p = R_i(t)$.

Example 5: Three soldiers on an OP have been instructed to put out 15 sensors forward of their OP to detect movement in a wooded area. They estimate that any movement through the area can be detected as long as at least 12 of the sensors are operating. Sensors are assumed to be in parallel (active redundant), i.e.: they fail independently. If we know that each sensor has a 0.6065 probability of operating properly for at least 24 hours, we can compute the reliability of the entire sensor system for 24 hours.

Define the random variable: X = number of sensors working after 24 hours.

Clearly, the random variable X is binomially distributed with n = 15 and p = 0.6065. In the language of mathematics, we write this sentence as

$$X \sim b(15{,}0.6065) \text{ or } X \sim \text{BINOMIAL}(15{,}0.6065).$$

We know that the reliability of the sensor system for 24 hours is

$$R(24) = P(X \geq 12) = P(12 \leq X \leq 15) = 0.0990.$$

Thus the reliability of the system for 24 hours is only 0.0990.

Modeling Standby Redundant Systems

Active redundant systems can sometimes be inefficient. These systems require only k of the n components to be operational, but all n components are initially in operation and thus subject to failure. An alternative is the use of spare components. Such systems have only k components initially in operation; exactly what we need for the whole system to be operational. When a component fails, we have a spare “standing by” which is immediately put in to operation. For this reason, we call these *Standby Redundant Systems*. Suppose our system requires k operational components and we initially have $n-k$ spares available. When a component in operation fails, a decision switch causes a spare or standby component to activate (becoming an operational component). The system will continue to function until there are less than k operational components remaining. In other words, the system works until $n-k+1$ components have failed. We will consider only the case where one operational component is required (the special case where $k=1$) and there are $n-1$ standby (spare) components available. We will assume that a decision switch (DS) controls the activation of the standby components instantaneously and 100% reliably. We use the model shown in figure 5 below to represent this situation.

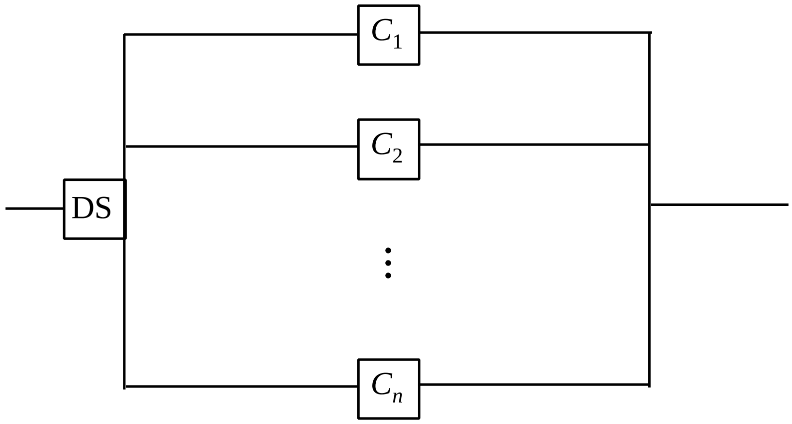


Figure 5: Standby Redundant System

If we let T_i be the time to failure of the *i*th component, then the T_i's are independent and identically distributed for $i = 1,2,3,\ldots,n$. Thus $R_i(t)$ is identical for all components. Let T = time to failure of the entire system. Since the system fails only when all n components have failed, and component $i+1$ is put into operation only when component i fails, it is easy to see that

$$T = T_1 + T_2 + \ldots + T_n .$$

In other words, we can compute the system failure time easily if we know the failure times of the individual components.

Finally, we can define a random variable

X = number of components that *fail* before time t in a standby redundant system.

Now the reliability of the system is simply equal to the probability that less than n components fail during the time interval (0, t). In other words,

$$R(t) = P(X < n).$$

It can be shown that X follows a Poisson distribution with parameter $l = a\,t$ where a is the failure rate, so we write $X \sim \text{POISSON}(l)$.

For example, if time is measured in seconds, then a is the number of failures per second. The reliability for some specific time t then becomes:

$$R(t) = P(X < n) = P(0 \le X \le n-1).$$

Example 6: Consider the reliability of a radio battery. We determined previously that one battery has a reliability for 24 hours of 0.4493. In light of the importance of communications, you decide that this reliability is not satisfactory. Suppose we carry two spare batteries. The addition of the spares should increase the battery system reliability. Later in the course, you will learn how to calculate the failure rate a for a battery given the reliability (0.4493 in this case). For now, we will give this to you: $a = 1/30$ per hour. We know that $n = 3$ total batteries. Therefore:

$$X \sim \text{POISSON}\left(l = a\ t = \frac{1}{30}(24) = 0.8 \right)$$

and

$$R(24) = P(X < 3) = P(0 \le X \le 2) = 0.9526.$$

The reliability of the system with two spare batteries for 24 hours is now 0.9526.

Example 7: If the OP must stay out for 48 hours without resupply, how many spare batteries must be taken to maintain a reliability of 0.95? We can use trial and error to solve this problem. We start by trying our current load of 2 spares. We have

$$X \sim \text{POISSON}\left(l = a\ t = \frac{1}{30}(48) = 16 \right),$$

and we can now compute the system reliability

$$R(48) = P(X < 3) = P(0 \le X \le 2) = 0.7834 < 0.95$$

which is not good enough. Therefore, we try another spare so $n = 4$ (3 spares) and we compute:

$$R(48) = P(X < 4) = P(0 \le X \le 3) = 0.9212 < 0.95$$

which is still not quite good enough, but we are getting close! Finally, we try $n = 5$ which turns out to be sufficient:

$$R(48) = P(X < 5) = P(0 \le X \le 4) = 0.9763 \ge 0.95.$$

Therefore, we conclude that the OP should take out at least 4 spare batteries for a 48 hour mission.

Models of Large Scale Systems

In our discussion of reliability up to this point, we have discussed series systems, active redundant systems, and standby redundant systems. Unfortunately, things are not always this simple. The types of systems listed above often appear as subsystems in larger arrangements of components that we shall call “large scale systems”. Fortunately, if you know how to deal with series systems, active redundant systems, and standby redundant systems, finding system reliabilities for large scale systems is easy. Consider the following example.

The first and most important step in developing a model to analyze a large scale system is to draw a picture. Consider the network that appears as figure 6 below. Subsystem *A* is the standby redundant system of three components (each with failure rate 5 per year) with the decision switch on the left of the figure. Subsystem B_1 is the active redundant system of three components (each with failure rate 3 per year), where at least two of the three components must be working for the subsystem to work. Subsystem B_1 appears in the upper right portion of the figure. Subsystem B_2 is the two component parallel system in the lower right portion of the figure. We define subsystem *B* as being subsystems B_1 and B_2 together. We assume all components have exponentially distributed times to failure with failure rates as shown in the figure.

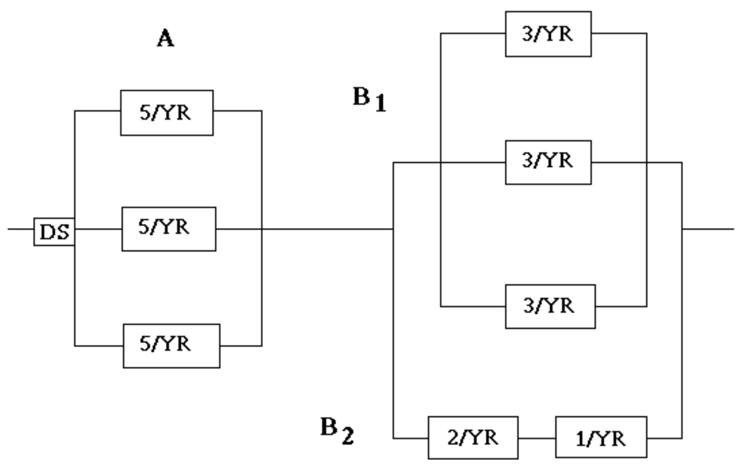


Figure 6: Network Example

Suppose we want to know the reliability of the whole system for 6 months. Observe that you already know how to compute reliabilities for the subsystems *A*, B_1, and B_2. Let's review these computations and then see how we can use them to simplify our problem.

Subsystem *A* is a standby redundant system, so we will use the Poisson model. We let

X = the number of components which fail in one year.

Since 6 months is 0.5 years, we seek $R_A(0.5) = P(X < 3)$ where *X* follows a Poisson distribution with parameter $\lambda = \alpha t = (5)(0.5) = 2.5$. Then,

$$R_A(0.5) = P(X < 3) = P(0 \le X \le 2) = 0.5438.$$

Now we consider subsystem B_1. In the section II, we learned how to find individual component reliabilities when the time to failure followed an exponential distribution. For subsystem B_1, the failure rate is 3 per year, so our individual component reliability is

$$R(0.5) = 1 - F(0.5) = 1 - \left(1 - e^{-(3)(0.5)}\right) = e^{-(3)(0.5)} = 0.2231.$$

Now recall that subsystem B_1 is an active redundant system where two components of the three must work for the subsystem to work. If we let

Y = the number of components that function for 6 months

and recognize that Y follows a binomial distribution with n = 3 and p = 0.2231, we can quickly compute the reliability of the subsystem B_1 as follows:

$$R_{B_1}(0.5) = P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y \leq 1) = 1 - 0.8729 = 0.1271.$$

Finally we can look at subsystem B_2. Again we use the fact that the failure times follow an exponential distribution. The subsystem consists of two components; obviously they both need to work for the subsystem to work. The first component's reliability is

$$R(0.5) = 1 - F(0.5) = 1 - \left(1 - e^{-(2)(0.5)}\right) = e^{-(2)(0.5)} = 0.3679\,,$$

and for the other component the reliability is

$$R(0.5) = 1 - F(0.5) = 1 - \left(1 - e^{-(1)(0.5)}\right) = e^{-(1)(0.5)} = 0.6065\,.$$

Therefore, the reliability of the subsystem is

$$R_{B_2}(0.5) = (0.3679)(0.6065) = 0.2231.$$

Our overall system can now be drawn as shown in figure 7 below.

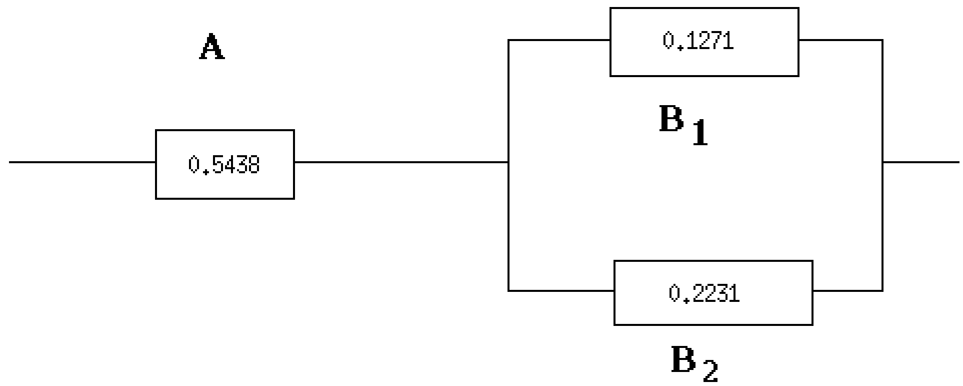


Figure 7: Simplified Network Example

From here we determine the reliability of subsystem *B* by treating it as a system of two independent components in parallel where only one component must work. Therefore,

$$R_B(0.5) = R_{B_1}(0.5) + R_{B_2}(0.5) - R_{B_1}(0.5) \cdot R_{B_2}(0.5)$$
$$= 0.1271 + 0.2231 - (0.1271)(0.2231) = 0.3218$$

Finally, since subsystems *A* and *B* are in series, we can find the overall system reliability for 6 months by taking the product of the two subsystem reliabilities:

$$R_{\text{system}}(0.5) = R_A(0.5) \cdot R_B(0.5) = (0.5438)(0.3218) = 0.1750 \text{ .}$$

We have used a network reduction approach to determine the reliability for a large scale system for a given time period. Starting with those subsystems which consist of components independent of other subsystems, we reduced the size of our network by evaluating each subsystem reliability one at a time. This approach works for any large scale network consisting of basic subsystems of the type we have studied (series, active redundant, and standby redundant).

We have seen how methods from elementary probability can be used to model military reliability problems. The modeling approach presented here is useful in helping students simultaneously improve their understanding of both the military problems addressed and the mathematics behind these problems. The models presented also motivate students to appreciate the power of mathematics and its relevance to today's military.

Exercises

1. A continuous random variable *Y*, representing the time to failure of a .50 cal machine gun tube, has a probability density function given by

$$f(y) = \begin{cases} \frac{1}{3}e^{\frac{-y}{3}} & y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the reliability function for *Y*.
b. Find the reliability for 1.2 time periods, *R*(1.2).

2. The lifetime of a HUMM-V engine (measured in time of operation) is exponentially distributed with a MTTF of 400 hours. You have received a mission that requires 12 hours of continuous operation. Your log book indicates that the HUMM-V has been operating for 158 hours.
a. Find the reliability of your engine for this mission.
b. If your vehicle's engine had operated for 250 hours prior to the mission, find the reliability for the mission.

3. A critical target must be destroyed. You are on the staff of the division G-3 when he decides to use helicopter gunships to destroy this key target. The aviation battalion is tasked to send four helicopter gunships. On their way to the target area, these helicopters must fly over enemy territory for approximately 15 minutes during which time they are vulnerable to anti-aircraft fire. The life of a gunship over this territory is estimated to be exponentially distributed with a mean of 18.8 minutes. It is further estimated that two or more gunships are required to destroy the target. Find the reliability of the gunships in accomplishing their mission (assuming the only reason a gunship fails to reach the target is enemy air defense systems).

4. For the mission in exercise 3, the division G-3 determines that, to justify risking the loss of gunships, there must be at least an 80% chance of destroying the target. How many gunships should the aviation battalion recommend be sent? Justify your answer.

5. Mines are a dangerous obstacle. Most mines have three components--the firing device, the wire, and the mine itself (casing). If any of these components fail, the systems fails. These components of the mine start to "age" when they are unpacked from their sealed containers. All three components have MTTF that are exponentially distributed of 60 days, 300 days, and 35 days, respectively.
a. Find the reliability of the mine after 90 days.
b. What is the MTTF of the mine?
c. What assumptions, if any, did you make?

6. You are a project manager for the new ADA system being developed in Huntsville, Alabama. A critical subsystem has two components arranged in a parallel configuration. You have told the contractor that you require this subsystem to be at least 0.995 reliable. One of the subsystems came from an older ADA system and has a known reliability of 0.95. What is the minimum reliability of the other component so that we meet our specifications?

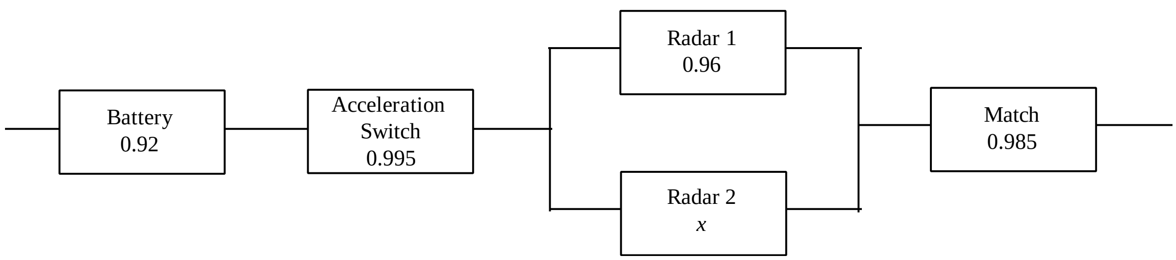
7. You are your battalion S-3. Your battalion has several night operations planned. There is some concern about the reliability of the lighting system for the Battalion's Tactical Operations Center (TOC). The lights are powers by a 1.5 KW generator that has a MTTF of 7.5 hours.

a. Find the reliability of the generator for 10 hours if the generator's reliability is exponential.

b. Find the reliability of the power system if two other identical 1.5 KW generators are available. First consider as active redundant and then as stand-by redundant. Which would improve the reliability the most?

c. How many generators would be necessary to insure a 99% reliability?

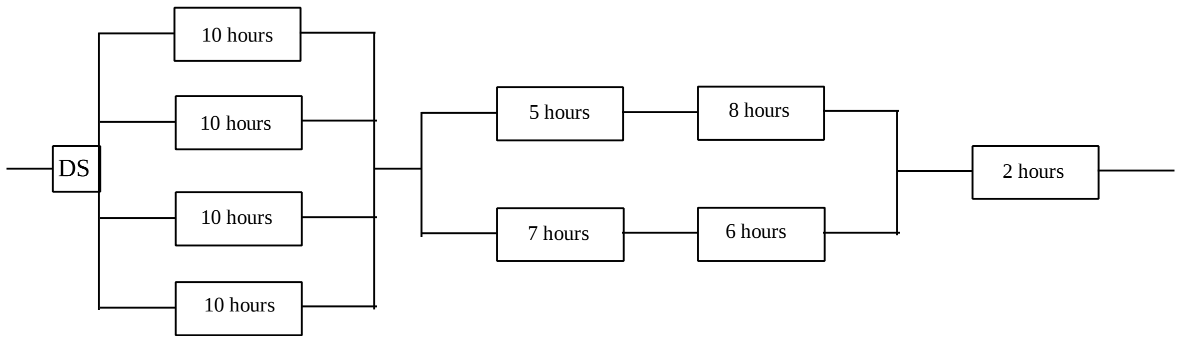
8. Consider the fire control system for a missile depicted below with the reliability for each component as indicated. Assume all components are independent and the radars are active redundant.



a. Find the system reliability for six months when $x = 0.96$.

b. Find the system reliability for six months when $x = 0.939$.

9. A major weapon system has components as shown in diagram below. All components have exponential times to failure with mean times to failure shown. All components operate independently of each other. Find the reliability for this weapon system for 2 hours.



10. Write an essay about how you can use elementary probability in modeling military problems.

References

[1] Devore, J. L., *Probability and Statistics for Engineers and Scientists, 4th Edition,* Duxbury Press, Belmont, CA (1995).

[2] Resnick, S. L., *Adventures in Stochastic Processes,* Birkhäuser, Boston (1992).