Arms Control and Warfare

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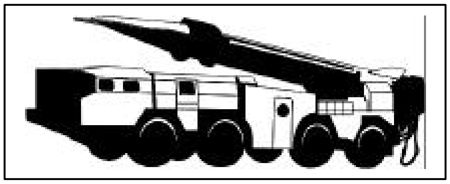
Introduction



What causes nations to wage war? History shows that the existence of weapons—large military arsenals—increases the likelihood of violent conflict. Without destructive weapons, perhaps nations sometimes would settle disputes by other means. It was this assumption that led Lewis Fry Richardson to begin his study and analysis of arms races. Richardson was a Quaker and was troubled by both WWI and WWII. His scientific training in physics led him to believe that wars were a phenomena that could be studied and mathematically modeled.

Richardson conjectured that arms races were often preludes to war. If nations were increasing their expenditures on defense budgets then a small spark could start a major conflagration. If two nations were decreasing their defense budgets, then a small incident might not trigger a war.

Ultimately, Richardson wanted to build a model to examine certain conditions in order to predict whether an arms race was “stable” or “unstable”.

The Arms Race Model

We examine the Richardson’s Arms Race Model initially as a system of linear difference equation— a system of discrete dynamical systems. We let,
X(n) = the armament of Nation X at time *t=n*.

The change in armament level from *t=n-1* to *t=n* is represented by:

$$DX(n) = X(n)-X(n-1) \qquad (1)$$

Simarly this model is also true for nation Y:
Y(n) = the armament of Nation Y at time *t=n*.

The change in armament level from *t=n-1* to *t=n* is represented by:

$$DY(n) = Y(n)-Y(n-1) \qquad (2)$$

Richardson envisioned the effects on each nation’s armament on the other nation. He added terms considering defense coefficients or how each nation is effected by the strength of the other nation

$$DX(n)= d_1 Y(n-1) \qquad (1a)$$

$$DY(n)= d_2 X(n-1) \qquad (2a)$$

Then he considered fatigue and expense coefficients of keeping up an arms race.

$$DX(n) = d_1 Y(n-1) - a_1 X(n-1) \qquad (1b)$$
$$DY(n) = d_2 X(n-1) - a_2 Y(n-1) \qquad (2b)$$

Finally, grievances or ambitions are added to the model as constants.

$$DX(n) = d_1 Y(n-1) - a_1 X(n-1) + g \qquad (1c)$$
$$DY(n) = d_2 X(n-1) - a_2 Y(n-1) + h \qquad (2c)$$

We call these final two equations (*1c)* and (*2c*), a system of discrete dynamical systems.

Estimates of the Model's Parameters

Consider the data in Table 1 for the arms build up in Iraq and Iran before their 1975 war. The data collected is the expenditures for arms by the two countries from 1954 to 1974. Let's use our model to analyze what occurred to cause this war to take place.

Year	Iran	Iraq
1954	78	75
1955	107	67
1956	126	94
1957	151	102
1958	243	110
1959	271	129
1960	292	145
1961	320	185
1962	345	206
1963	387	271
1964	425	359
1965	435	402
1966	460	450
1967	473	480
1968	498	513
1969	534	549
1970	612	723
1971	732	781
1972	840	921
1973	980	1292
1974	1308	1632

TABLE 1. Defense Expenditures for Iran and Iraq (1954-1974).

We use multiple linear regression to estimate the parameters of our model. We let *X(n)* stand for the defense expenditures for Iran in time period *n*. Similary, we let *Y(n)* stand for the defense expenditures for Iraq in time period *n*. We regress *X(n)*—the response variable on the predictors—*X(n-1)* and *Y(n-1)*. We also regress *Y(n)* on its two predictors—*Y(n-1)* and *X(n-1)*. Using MINITAB to perform the multiple linear regression models, we achieve the following results (MINITAB printout):

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Worksheet size: 100000 cells

MTB > Regress c5 2 c2 c3;
SUBC>   Constant.

Regression Analysis
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The regression equation is
X(n) = 37.1 + 0.651 X(n-1) + 0.432 Y(n-1)

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20 cases used 1 cases contain missing values

Predictor       Coef       StDev          T        P
Constant       37.06       26.35       1.41    0.178
X(n-1)        0.6508      0.1651       3.94    0.001
Y(n-1)        0.4317      0.1204       3.59    0.002

S = 38.91       R-Sq = 98.5%     R-Sq(adj) = 98.3%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    2     1689279      844639    557.84    0.000
Error        17       25740        1514
Total        19     1715019

Source       DF      Seq SS
X(n-1)       1     1669816
Y(n-1)       1       19463

Unusual Observations
Obs       X(n-1)  X(n)    Fit  StDev Fit   Residual    St Resid
 20       980    1308.00    1232.56      28.56      75.44       2.85RX
 21      1308          *    1592.79      34.58          *           * X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

MTB > Regress c6 2 c2 c3;
SUBC>   Constant.

Regression Analysis
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The regression equation is
Y(n) = - 52.9 + 0.195 X(n-1) + 1.13 Y(n-1)

```
20 cases used 1 cases contain missing values

Predictor       Coef       StDev          T        P
Constant      -52.91       40.06      -1.32    0.204
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X(n-1)	0.1949	0.2510	0.78	0.448
Y(n-1)	1.1268	0.1830	6.16	0.000

S = 59.15 R-Sq = 98.2% R-Sq(adj) = 98.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	3337341	1668671	476.90	0.000
Error	17	59484	3499		
Total	19	3396825			

Source	DF	Seq SS
X(n-1)	1	3204724
Y(n-1)	1	132617

Unusual Observations

Obs	X(n-1)	Y(n)	Fit	StDev Fit	Residual	St Resid
19	840	1292.0	1148.6	28.2	143.4	2.76R
20	980	1632.0	1593.9	43.4	38.1	0.95 X
21	1308	*	2041.0	52.6	*	* X

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Extracting the regression equations from the MINITAB output, we get the coupled nonhomogeneous system model as:

$$X(n) = 37.1 + 0.651\, X(n-1) + 0.432\, Y(n-1) \qquad (3)$$

$$Y(n) = -52.9 + 0.195\, X(n-1) + 1.13\, Y(n-1) \qquad (4)$$

Model Solution and System Long Term Behavior (Stability Analysis)

Using linear algebra, we can solve for the stability of the system. The model (in matrix form with *Iran(n) = X(n)* and *Iraq(n) = Y(n)*) is:

$$\begin{bmatrix} Iran(n) \\ Iraq(n) \end{bmatrix} = \begin{bmatrix} .651 & .432 \\ .195 & 1.13 \end{bmatrix} \begin{bmatrix} Iran(n-1) \\ Iraq(n-1) \end{bmatrix} + \begin{bmatrix} 37.1 \\ -52.9 \end{bmatrix} \qquad (5)$$

Let A(n) be the vector representing $\begin{bmatrix} Iran(n) \\ Iraq(n) \end{bmatrix}$ and A(n-1) be the vector representing $\begin{bmatrix} Iran(n-1) \\ Iraq(n-1) \end{bmatrix}$. The model can now be written, more easily, as

$$A(n) = \begin{bmatrix} .651 & .432 \\ .195 & 1.13 \end{bmatrix} A(n-1) + \begin{bmatrix} 37.1 \\ -52.9 \end{bmatrix}. \qquad (6)$$

Finding and using eignevalues and eignevectors, we obtain the following solution to the homogeneous part of the system:

$$A(n) = c_1(1.266798)^k \begin{bmatrix} .70152 \\ 1 \end{bmatrix} + c_2(.5142019)^k \begin{bmatrix} 1 \\ -.31662 \end{bmatrix}$$

(7)

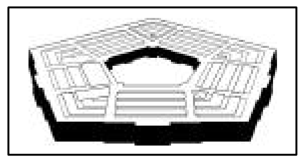
We use the formula (or conjecture) $D=(I-R)^{-1}B$ to find the nonhomogeneous part of the solution.

$$D = \begin{bmatrix} .349 & -.432 \\ -.195 & -.13 \end{bmatrix}^{-1} \begin{bmatrix} 37.1 \\ -52.9 \end{bmatrix} = \begin{bmatrix} 213.531 \\ 86.626 \end{bmatrix}. \quad (8)$$

The final general solution is

$$A(n) = c_1(1.266798)^k \begin{bmatrix} .70152 \\ 1 \end{bmatrix} + c_2(.5142019)^k \begin{bmatrix} 1 \\ -.31662 \end{bmatrix} + \begin{bmatrix} 213.531 \\ 86.626 \end{bmatrix}. \quad (9)$$

As $k \to \infty$, the term $(1.266798)^k$ grows without bound. This system is not stable. Thus, this is an unstable system and is conducive to war.



A stable arms race would indicate that there is at least one equilibrium point where both nations are satisfied. There is no need to escalate armament build up beyond this equilibrium point. The equilibrium point in an arms race represents the level of arms such that the dynamics of the arms race ceases. It is the value of the armaments that results in no need to change the armaments of the two nations. An unstable arms race indicates that no equilibrium point exists. The expenditures continue to escalate as does the build up of destructive weapons. The dynamics of the arms race continues as any positive change in armaments in one nation results in a positive change in armaments to the other nation. Perhaps a small spark or act can trigger a conflict in this unstable case.

Exercises

1. Find the particular solution to the Iran-Iraq arms race model if the initial conditions are Iran(0)=78 and Iraq(0)=75.

2. Write a short essay on the nature of war and how eigenvalues can help to determine the stability of the arms race.

3. Given the following data for the arms race between the Warsaw Pact forces and the NATO forces, use the Richardson's Arms Race model to

(a) Estimate the parameters for the NATO-Warsaw Forces

(b) Solve the model

(c) Determine the stability of the arms race

(d) Write a short essay concerning your modeling result and the reality of the 1980-90's scenario in Eastern Europe.

Year	NATO	WTO
1971	206.1	166.6
1972	209.6	173.9
1973	205.6	180.9
1974	208.6	188.5
1975	206.1	195.3
1976	202.8	203.8
1977	209.9	206.9
1978	212.7	210.1
1979	218.8	212.6
1980	229.8	218.9

<u>References</u>

1. Fox, William P., Frank Giordano, and Maury Weir. *A First Course in Mathematical Modeling*, Brooks/Cole Publishing. Pacific Grove, CA. 1997.

2. Schrodt, Phillip. *Richardson's Arms Race Model*. National Collegiate Software Clearinghouse. Raleigh, NC. 1987.

3. Zinnes, Dina A. John Gillespie, and G.S. Tahim. *The Richardson's Arms Race Model, UMAP Module 308*. COMAP. Boston, MA. 1990.