设$\{a_n\}$是一个数列，并根据式(1)定义数列$\{S_n\}$:

$$S_n = \sum_{k=1}^{n} a_k\ ,\quad n = 1, 2, 3, \cdots \tag{1}$$

已知极限$\lim\limits_{n\to+\infty} S_n$存在，按照式(2)定义$S \in \boldsymbol{C}$:

$$S = \lim_{n\to+\infty} S_n \tag{2}$$

设数列$\{\overline{S}_n\}$满足式(3), (4)和(5):

$$S_n = \overline{S}_n + a \cdot q^n \tag{3}$$

$$S_{n+1} = \overline{S}_n + a \cdot q^{n+1} \tag{4}$$

$$S_{n+2} = \overline{S}_n + a \cdot q^{n+2} \tag{5}$$

其中: $a \in \boldsymbol{C}$ 和 $q \in \boldsymbol{C}$ 都是待定的系数.

由式(3), (4)和(5)可以得到式(6):

$$\frac{S_{n+2} - S_{n+1}}{S_{n+1} - S_n} = \frac{\overline{S}_n + a \cdot q^{n+2} - \overline{S}_n - a \cdot q^{n+1}}{\overline{S}_n + a \cdot q^{n+1} - \overline{S}_n - a \cdot q^n} = \frac{a \cdot q^{n+2} - a \cdot q^{n+1}}{a \cdot q^{n+1} - a \cdot q^n}$$

$$q = \frac{S_{n+2} - S_{n+1}}{S_{n+1} - S_n} \tag{6}$$

将式(1)代入式(6)，可以得到式(7):

$$q = \frac{\sum\limits_{k=1}^{n+2} a_k - \sum\limits_{k=1}^{n+1} a_k}{\sum\limits_{k=1}^{n+1} a_k - \sum\limits_{k=1}^{n} a_k}$$

$$q = \frac{a_{n+2}}{a_{n+1}} \tag{7}$$

由式(3)和(4)可以得到式(8):

$$S_{n+1} - S_n = \overline{S}_n + a \cdot q^{n+1} - \overline{S}_n - a \cdot q^n = a\left(q^{n+1} - q^n\right)$$

$$a = \frac{S_{n+1} - S_n}{q^{n+1} - q^n} \tag{8}$$

将式(1)和(7)代入式(8)，得到式(9):

$$a = \left[\left(\sum_{k=1}^{n+1} a_k\right) - \left(\sum_{k=1}^{n} a_k\right)\right] \Bigg/ \left(\frac{a_{n+2}^{n+1}}{a_{n+1}^{n+1}} - \frac{a_{n+2}^{n}}{a_{n+1}^{n}}\right)$$

$$a = \frac{a_{n+1}^{n+2}}{a_{n+1}^{n+2} - a_{n+1}\, a_{n+2}^{n}} \tag{9}$$

将式(7)和(9)代入式(3)，得到式(10):

$$S_n = \overline{S}_n + \frac{a_{n+1}^{n+2}}{a_{n+1}^{n+2} - a_{n+1}\, a_{n+2}^{n}} \cdot \frac{a_{n+2}^{n}}{a_{n+1}^{n}} = \overline{S}_n + \frac{a_{n+1}^{2}}{a_{n+2} - a_{n+1}}$$

$$\overline{S}_n = S_n - \frac{a_{n+1}^{2}}{a_{n+2} - a_{n+1}}\ ,\quad n = 1, 2, 3, \cdots \tag{10}$$

式(10)给出了数列$\{\overline{S}_n\}$的通项公式.

如果极限$\lim\limits_{n\to+\infty} \overline{S}_n$存在，那么随着$n$的增大，$\overline{S}_n$可能比$S_n$更快收敛.

将数列$\{\overline{S}_n\}$看作是另一个数列的部分和，并将这一数列记为$\{\overline{a}_n\}$，那么根据部分和的定义可以得到式(11)和(12):

$$\overline{a}_n = \begin{cases} \overline{S}_1 & , \quad \text{if } n = 1 \\ \overline{S}_n - \overline{S}_{n-1} , & \text{if } n \geq 2 \end{cases} \tag{11}$$

$$\overline{S}_n = \sum_{k=1}^{n} \overline{a}_k \ , \quad n = 1, 2, 3, \cdots \tag{12}$$

按照式(13)定义数列$\left\{\hat{S}_n\right\}$:

$$\hat{S}_n = \overline{S}_n - \frac{\overline{a}_{n+1}^{\ 2}}{\overline{a}_{n+2} - \overline{a}_{n+1}} , \quad n = 1, 2, 3, \cdots \tag{13}$$

如果极限 $\lim\limits_{n \to +\infty} \hat{S}_n$ 存在, 那么随着 n 的增大, $\hat{S}_n$ 可能比 $\overline{S}_n$ 更快收敛.