

A FAST SORTING ALGORITHM TO REDISTRICTING ELECTION MODEL

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Abstract : In This paper, we study the problem of redistricting election. Then a redistricting election model is proposed and an aim directed algorithm is established to cope with the model through fairness principles and the breadth first search method. Theoretical analysis and numerical experiment indicate that the model is reasonable and the algorithm is quite feasible.

Keywords: Gerrymander; Election; aim directed algorithm; voters-seats curve

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1 Introduction

In the history of parliament election in US, the word "Gerrymander" originated in 1812, when the governor of Massachusetts, Elbridge Gerry want to win in the election, so he deliberately redistricted constituency and made the hostile party candidate votes concentrated in a few constituencies, in order to protect his party's candidate was elected disproportionately. Gerrymander indicated dividing the election district in an unfair way for political purposes. Gerrymander had two means of operation: centralized and decentralized ballot votes. The concentration of votes refers to dividing supporters of a party into the area where another party has secured the election, resulting in a waste of the party's ballot; scattered votes refers to that the voters who support a party will be splitted into different constituencies, so that the constituency was not able to concentrate enough votes to elect the candidates they support.

Therefore, in order to avoid " Gerrymander " phenomenon and ensure the fairness of the elections, the United States passed legislation which explicitly requests taking a census of population every ten years, registering voter public opinion, redistricting constituencies and asking that constituencies are closely linked. Lijphart presented 16 criterias which were used to regulate the principle of division of constituencies. The purpose of this paper is to design a simple and feasible model for the division of constituencies to ensure the fairness of the election and redistrict the constituency automatically through the algorithm.

2 Redistricted model for fair elections

Suppose County is the smallest administrative unit, division of constituencies is to divide one or a few neighboring counties into a single constituency. For division of

constituencies ,the most important is to ensure fairness, Fairness is one person,one vote to the voters ,and to the partisan it is that the partisan which had more supporters would get more seats in the election.

2.1 Equalization of the population

In order to ensure one person, one equal vote, each small constituency need to ensure equalization of the population.

Suppose p_i represents the population of i county, P_i represents the total population of i th constituency , so

$$P_i = \sum_{j=1}^N p_j \delta_{ij}$$

Here, $\delta_{ij} = \begin{cases} 1 & s_i = j \\ 0 & s_i \neq j \end{cases}$, when $s_i = j$, it said that i county is divided into j th

constituency. Suppose that there are a total of m constituencies, the total population of all the constituencies is,

$$P = \sum_{i=1}^m P_i$$

The average population of each constituency is $P_a = P/m$

If you want to make an absolute guarantee of equalization of the population, we need to make $P_i = P_a$ for any constituency, but taking into account the practical problems, the difference is difficult to avoid, it needs to allow a certain degree of bias in the actual operation, the definition of the population deviation function is as follows

$$A_p = \sum_{i=1}^m \left| 1 - \frac{P_i}{P_a} \right|$$

Take the specific constant A , when $A_p = A$, the equalization of the population are met.

2.2 Connectivity

In the division of constituencies, it has the need to ensure the connectivity of every constituencies that each constituency should be a combination of linked counties, but we need to avoid the re-connected situation ,it is said that a constituency can't surrounded completely another constituency .

2.3 Compactness

Constituency compactness is an important tool to prevent "Gerrymander", since constituency compactness is to ask the electoral constituencies shape to meet certain rules, and not to appear grotesque, for example, it appears slender shape of constituencies. The compactness function is defined as follows: Suppose r is equal to the distance from a certain point to the central point in the region Ω , so its moment of inertia is: $\iint_{\Omega} r^2 d\Omega$, this value describes the level of the points around closer to the center point. Suppose D represents the area of region, the definition of compactness function is

$$M = \frac{D}{\sqrt{4\pi} \iint_{\Omega} r^2 d\Omega} \quad (2)$$

Thus, if Ω is a circular area, then, M is equal to 1, which is exactly the shape of an ideal constituency.

2.4 Division of constituencies

Fairness in the division of constituencies, as the above, it had given the limit through the equalization of the population, connectivity and compactness, in order to reflect the fairness of the constituencies division to parties, in each constituencies, mechanisms need to give an equitable relationship between a vote and the elected seats in order to prevent appearing a waste of the votes on purpose. Suppose V represents the votes rate that a certain political party gained, $f(V)$ represents the party gained the seats rate, in single-member district system, the most equitable relationship is $f(V)=V$. But in multi-member constituency, the situation is more complicated.

First of all, fair $f(V)$ should meet the following two properties:

(1) $f(V) \in [0,1]$, and through the three points (0, 0), (0.5, 0.5), (1, 1), the meaning is clear, when a party gained 0 votes, it should not get seats, when it gains all the votes, it should get all the seats, when a party and other parties are evenly matched, the number of seats should be equal.

(2) $f(V)$ is an increasing function, indicating when a party gets more votes rate it should be given more seats.

Secondly, we adopt such a strategy, in a constituency, if a party has a majority of the votes rate, we allow it to get lower seats rate than the votes rate, such an approach can increase the competitiveness of small parties, therefore, the purpose of doing so can make the division of constituencies on the behavior tend to make votes rate between the parties equal, thus avoiding the centralized or decentralized ballot votes to a certain extent. If you do not do so, it would easily lead to the deterioration of centralized or decentralized votes, but also lead to some other social problems.

Therefore, taking the above analysis into account, introduce the *bi*logit functional form

$$f(V) = \{1 + e^{-\lambda - \ln(\frac{V}{1-V})}\}^{-1} \quad (3)$$

λ expressed party bias parameter, $\lambda = 0$ expressed non-partisan bias, that is, $f(V)=V$, $\lambda > 0$ expressed a bias for big parties, that is, the party which had a high votes rate could get higher seats rate than votes rate; $\lambda < 0$ expressed a bias for small parties, that is to say the party which had low votes rate could get higher seats rate than votes rate.

3 Aim directed algorithm

Through the above model, we defined some criterias for fairness of the division of constituencies, the following task is how to achieve the division of constituencies through the algorithm under the circumstance of meeting fairness model, that is, a combination of

different counties to complete the division of constituencies. Our algorithm's strategy is to take a dynamic merger, that is, from the initial area (such as a county), search neighboring county on condition that meeting the model, when meeting the constituency conditions, the division of constituencies completed, and so on until all regions were divided up.

Consider the two-party election, first of all, suppose V_A, V_B respectively representing the votes rate of parties A and B, S_A, S_B respectively representing the seats rate they get, M is on behalf of compactness evaluation value. Algorithm steps are as follows:

Step 1 To find the minimum convex polygon in the map of state, due to shapes of the maps of each state and county are not regular, the sizes are difficult to calculate, so we use its area of the minimum convex polygon instead;

Step 2 Calculate the average population of a constituency P_a , select the minimum convex polygon vertices where the county is as the initial search point i , and take the county as the first constituency I where the county is;

Step 3 Mark the population of the initial constituency $P_I = p_i$, calculate the current V_A, V_B, S_A, S_B and M , while searching the nearby counties j and joining it to the constituency;

Step 4 Determine whether the county j meet the the model conditions or not, if meeting the model equation (1) (2) (3), join j to the constituency I , at the same time make an amendment to the population value $P_I = p_i + p_j$; If do not meeting the conditions, select the other points next to i ;

Step 5 when all the counties have been searched, but still unable to complete the division of constituency work, then transfer to Step 2 to re-search the initial point and search until the constituency meet the condition;

Step 6 Exclude the county which has completed the division of constituencies, turn to Step 1, proceed to the next constituency division;

Step 7 If all the counties have been divided, the algorithm stops.

This algorithm can reflect the effect of mode. Algorithm in the search process is always searching for a near point, so the connectivity of constituency is guaranteed; when the population meet the requirements, the division of constituency ends, equalization of the population is guaranteed, while the calculation of V_A, V_B, S_A, S_B and

M is most important, they reflect the constituency fairness. When a county joins to the constituency, whether it undermines the principle of fairness or not, give assessment on the adoption of these values. At the same time algorithm has some certain risks, when the district division is completed, there is a isolated county, when considering the initial search point for more than one, it maybe appear a mad scramble for a county case, that is, the current constituencies all require including a particular county to complete division

of constituencies, when there are these two situations, the algorithm need to resume. The core step of algorithm is to search neighboring point, it is the breadth-first search, therefore, complexity is $O(n^2)$.

4 Numerical experiments

We use the 2004 New York State's election results as preliminary data. There are 29 seats for New York, the United States according to the U.S.Constitution. There are 29 district-based constituencies in 2004. The results which are figured out according to goal directed algorithm are shown in the Figure 1. Other information, such as population, area, compactness in Table 1 (only shows the top three areas). According to the sheet 1, we can draw a conclusion that different constituencies have different shapes, at the same time, the shapes of different constituencies are feasible, the compact evaluation value is closer to the ideal graph. Therefore, the compact evaluation value of this article can be trusted.

Table 1 The number of voters in constituency , area , information table of close degree

表 1 选区选民数、面积、紧密度信息表			
选 区	选民数	面 积	紧密度
1	417348	2785	1.0209
2	41000	631	0.9885
3	392752	5056	1.1989

5 Conclusion

This article aims at democratic elections, establishes a definition and standard which is in relation to many fields, transform the political issue into a mathematical model and designs a polynomial-time algorithm(or directional target algorithm)is used to solve the model. By studying the division of constituencies of New York State , it indicates the model designed in this paper is reasonable and the algorithm is effective and applicable.

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